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Crossover in randomly diluted classical two-dimensional Heisenberg magnets

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Abstract. We use a truncated replicated nonlinear sigma model to study crossover in the vicinity of the percolation threshold of a randomly diluted classical m-component Heisenberg model with $m > 2$. We find that the order parameter correlation length ξ diverges with temperature T for concentration p of occupied bonds greater than the percolation threshold p_c as $\exp(2\pi\rho_s(p)/T)$ where $\rho_s(p) \sim |p - p_c|^{\phi}$ is the spin stiffness and for $p = p_c$ as $T^{-\phi/\nu_p}$ where ν_p is the percolation correlation length exponent. The numerical values of the exponents ϕ and ν_p predicted in our truncation scheme, however, differ significantly from their accepted values.

1. Introduction

A randomly diluted m-component Heisenberg model with O_m symmetry undergoes a percolation transition (Stauffer 1975, Lubensky 1975, Stanley *et al* 1976) as a function of the concentration p of occupied sites or bonds. For p greater than the percolation threshold p_c, there is an infinite connected cluster of spins and a non-zero magnetisation at temperature $T = 0$. At finite temperature, there is a thermally driven phase transition to a paramagnetic state for all spatial dimension $d > 2$ with a transition temperature $T_c(p)$ that goes continuously to zero as $p \to p_c$ as shown in figure 1(a). The point

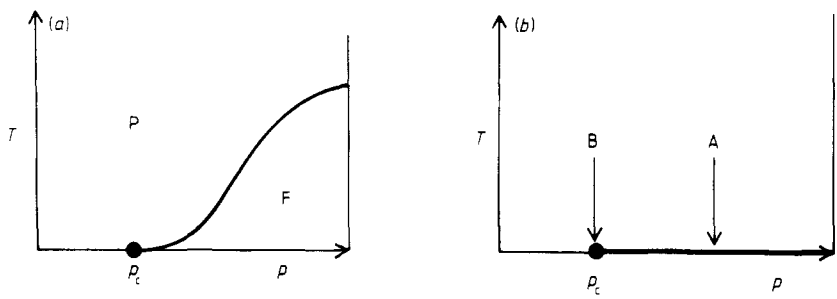

Figure 1. (a) Phase diagram of a randomly diluted O_m model in spatial dimension $d > 2$. There is no long-range order at any temperature until there is an infinite cluster of connected spins for concentration of occupied bonds p greater than the percolation threshold p_c. For $p > p_c$, there is a ferromagnetic phase (F) with a transition temperature to the paramagnetic phase (P) that goes to zero at p_c as $|p - p_c|^{\phi}$. (b) Phase diagram for the randomly diluted O_m model with $m > 2$ in two dimensions. There is long-range order only at $T = 0$ for $p > p_c$. The order parameter correlation length ξ diverges as $T \to 0$ as $\exp(2\pi\rho_s(p)/T)$ for $p > p_c$ (path A) and as $T^{-\phi/\nu_p}$ at $p = p_c$ (path B).

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$p = p_c$, $T = 0$ is thus a multicritical point at which both $p - p_c$ and T are relevant variables. The scaling exponent associated with $p - p_c$ is the inverse of the percolation correlation length exponent ν_p. The exponent $\lambda_t = \phi/\nu_p$ associated with temperature is related to the exponent t controlling the growth of the spin wave stiffness (Stephen 1978, Harris and Lubensky 1987) ρ_s or the conductivity Σ (Straley 1976, de Gennes 1976, Skal and Shklovskii 1976, Harris and Fisch 1977) of a randomly diluted resistor network via

$$t = (d-2)\nu_p + \phi. \tag{1.1}$$

Thus, in the vicinity of the percolation multicritical point, the spin correlation length $\xi(T, p)$ obeys the scaling relation

$$\xi(T, p) = |\Delta p|^{-\nu_p} f(T/(\Delta p)^{\phi}) \tag{1.2}$$

where $\Delta p = p - p_c$. This equation implies that

$$\xi \sim T^{-\nu_p/\phi} \tag{1.3}$$

at $p = p_c$ and that

$$T_c(p) \sim (\Delta p)^{\phi} \tag{1.4}$$

for $d > 2$.

In exactly two dimensions, undiluted O_m models (Polyakov 1975, Migdal 1976a, b, Brezin and Zinn-Justin 1975, Pelcovitz and Nelson 1976, Chakravarty *et al* 1988) with $m > 2$ have no long-range or quasi-long-range order except at $T = 0$: the paramagnetic phase is the stable phase for all $T > 0$. At $T = 0$, the ferromagnetic state is characterised by a spin wave stiffness ρ_s. The correlation length in the paramagnetic phase diverges exponentially rather than as a power of temperature:

$$\xi \sim \exp(2\pi\rho_s/T). \tag{1.5}$$

As the system is diluted, it remains ferromagnetic at $T = 0$ for $p > p_c$ as shown in the phase diagram of figure 1(b). The spin wave stiffness $\rho_s(p)$ decreases as p decreases to zero as $(p - p_c)^{\phi}$ (Harris and Lubensky 1987), even in two dimensions. One therefore expects the correlation length in the diluted magnet to diverge exponentially with temperature but with a renormalised exponent determined by $\rho_s(p)$:

$$\xi(T, p) \sim \exp(2\pi\rho_s(p)/T). \tag{1.6}$$

Since the exponent $\nu_p = \frac{4}{3}$ (den Nijs 1979, Nienhuis 1982) and $\phi = 1.296$ (Zabolitzky 1984, Lobb and Frank 1984, Herman *et al* 1984, Hong *et al* 1984) are perfectly regular in two dimensions, the spin correlation length at $p = p_c$ diverges as the power law $T^{-\nu_p/\phi}$ (equation (1.3)) even in two dimensions.

The nature of the crossover from exponential to power-law divergence of the correlation length has not, to our knowledge, been investigated and is the subject of this paper. We will use a replicated random nonlinear sigma model (Polyakov 1975, Migdal 1976a,b, Brezin and Zinn-Justin 1976, Pelcovitz and Nelson 1976) to study this crossover in the vicinity of the percolation threshold of a classical m-component Heisenberg model with $m > 2$. Our analysis, like that of Chakravarty *et al* (1988) is based upon one-loop renormalisation group recursion relations and is only approximate. The potentials entering our replicated field theory are the cumulants of the exchange with respect to the probability distribution of random dilution. There are an infinite number of these cumulants, and they are all needed to obtain an exact description of the model. At the pure, fully ordered, zero-temperature fixed point, the

potentials associated with the kth cumulant are irrelevant for $k \geqslant 2$ and become more irrelevant with increasing k. We arbitrarily truncate the full replicated field theory, retaining only the most relevant first and second cumulants of the exchange. The first cumulant or average exchange provides a measure of the inverse temperature. There are two potentials of different symmetry arising from the second cumulant. In the space of these two potentials at zero temperature, we find three fixed points as shown in figure 2 in addition to the fixed point of the undiluted system, which is stable with respect to randomness in agreement with the results of Murthy (1988). Of the three new fixed points, two are doubly unstable and in unphysical regions of parameter space. We identify the remaining singly unstable fixed point as the percolation fixed point. The correlation length divergence in the vicinity of this fixed point is a power law in temperature at $p = p_c$ and exponential in temperature of $p > p_c$. The numerical values of the percolation correlation length and crossover exponents obtained in our approximate analysis are respectively $\nu_p = \frac{1}{2}$ and $\phi = 0.195$. Unfortunately, these are quite far from the accepted value listed above. Thus, though the predictions of our one-loop truncated field theory are in qualitative agreement with what one would expect, they are not in good quantitative agreement with known results.

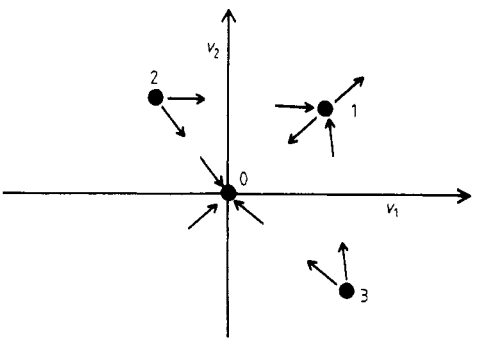

Figure 2. Renormalisation group flow diagram in the v_1–v_2 plane at $T = 0$. There are four fixed points: (0) the stable ordered-phase fixed point with $v_1 = v_2 = 0$, (1) the once-unstable fixed point in the physical first quadrant, which we identify with the percolation fixed point, and two twice-unstable fixed points (2) and (3) in the unphysical second and fourth quadrants.

The percolation fixed point in our analysis is very similar to the RVB fixed point of the quantum nonlinear sigma model for an antiferromagnet discussed by Chakravarty *et al* (1988). This model is characterised by a parameter y, in addition to the temperature, proportional to $\hbar$. The RVB fixed point separates the small-y ordered Neél state from a large-y melted spin fluid, which has been identified (Chakravarty *et al* 1988) with some version of the resonating valence bond (RVB) state (Anderson 1987, Kivelson *et al* 1987). As a function of temperature, the correlation length diverges exponentially with a quantum renormalised spin stiffness as the Neél state is approached and with a power law as the RVB critical point is approached. Dilution in the classical spin model plays the same role as quantum fluctuations in the quantum model. In both cases, the spin wave stiffness goes to zero at a critical point at which the correlation length diverges as a power law in temperature.

2. The model

We begin with a classical O_m model with unit length spins $\boldsymbol{S}(\boldsymbol{x})$ at each site $\boldsymbol{x}$ on a d-dimensional lattice with coordination number z and lattice spacing a. The spins interact via an exchange $j(\boldsymbol{x}, \boldsymbol{x})$ depending on the bond $\langle \boldsymbol{x}, \boldsymbol{x} \rangle$. The reduced Hamiltonian is

$$H = \frac{1}{T} \sum J(\boldsymbol{x}, \boldsymbol{x}')[\boldsymbol{S}(\boldsymbol{x}) - \boldsymbol{S}(\boldsymbol{x}')]^2 \tag{2.1}$$

where T is the temperature. The exchange is a quenched random variable, and we introduce replicated spins (Edwards and Anderson 1975) $\boldsymbol{S}^\alpha(\boldsymbol{x})$ with $|\boldsymbol{S}^\alpha(\boldsymbol{x})|^2 = 1$ for $\alpha = 1, \ldots, n$ to obtain the quenched averaged free energy via the replicated partition function

$$[Z^n]_{\text{av}} = \text{Tr} \exp\left[- \sum_{\langle \boldsymbol{x}, \boldsymbol{x}' \rangle} \bar{H}_{\boldsymbol{x}, \boldsymbol{x}'} \right] \tag{2.2}$$

where $[\ \ ]_{\text{av}}$ indicates an average over $J(\boldsymbol{x}, \boldsymbol{x}')$ and

$$\begin{aligned} H_{\boldsymbol{x}, \boldsymbol{x}'} &= -\ln\left[\exp\left(T^{-1} J(\boldsymbol{x}, \boldsymbol{x}') \sum_\alpha [\boldsymbol{S}^\alpha(\boldsymbol{x}) - \boldsymbol{S}^\alpha(\boldsymbol{x}')]^2 \right) \right]_{\text{av}} \\ &= \bar{K} \sum_\alpha [\boldsymbol{S}^\alpha(\boldsymbol{x}) - S^\alpha(\boldsymbol{x}')]^2 - \tfrac{1}{2}\Delta \left(\sum_\alpha [\boldsymbol{S}^\alpha(\boldsymbol{x}) - \boldsymbol{S}^\alpha(\boldsymbol{x}')]^2 \right)^2 \\ &\quad - \frac{1}{3!} \Delta_3 \left(\sum_\alpha [\boldsymbol{S}^\alpha(\boldsymbol{x}) - \boldsymbol{S}^\alpha(\boldsymbol{x}')]^2 \right)^3 + \ldots \end{aligned} \tag{2.3}$$

where

$$\bar{K} = [J]_{\text{av}}/T \qquad \Delta = ([J^2]_{\text{av}} - [J]_{\text{av}}^2)/T^2 \tag{2.4}$$

and Δ_k, $k = 3, \ldots$ is the kth cumulant of J/T. For a randomly diluted system, $J(\boldsymbol{x}, \boldsymbol{x}')$ is equal to J with probability p and zero with probability $(1-p)$. In this case

$$H_{\boldsymbol{x}, \boldsymbol{x}'} = -\ln\left[1 - p + p \exp\left((J/T) \sum_\alpha [S^\alpha(\boldsymbol{x}) - S^\alpha(\boldsymbol{x}')]^2 \right) \right] \tag{2.5}$$

and

$$\bar{K} = pJ/T \qquad \Delta = p(1-p)J^2/T^2. \tag{2.6}$$

Note that $\bar{K}$ remains non-zero for all $0 < p < 1$; its inverse provides a measure of the temperature.

We now set $\boldsymbol{S}^\alpha = (\boldsymbol{\pi}^\alpha, \sigma^\alpha)$ with $\boldsymbol{\pi}^\alpha = (\pi_1^\alpha, \ldots, \pi_{m-1}^\alpha)$ and $(\boldsymbol{\pi}^\alpha)^2 + (\sigma^\alpha)^2 = 1$, take the continuum limit of (2.2), and rescale lengths via $\boldsymbol{x} \to \Lambda \boldsymbol{x}$ where Λ is a wavenumber cutoff of order $2\pi/a$. The resulting replicated partition function is

$$[Z^n]_{\text{av}} = \int \mathrm{D}\pi \, \mathrm{e}^{-\bar{H}} \tag{2.7}$$

where

$$\bar{H} = \frac{1}{2t}\sum_{\alpha}\int \mathrm{d}^d x[(\nabla \boldsymbol{\pi}^{\alpha})^2 + (\nabla \sigma^{\alpha})^2]$$

$$- t^{-2} v_{ijkl} \sum_{\alpha\beta} \int \mathrm{d}^d x(\nabla_i \pi_a^{\alpha} \nabla_j \pi_a^{\alpha} \nabla_k \pi_b^{\beta} \nabla_l \pi_b^{\beta} + \nabla_i \sigma^{\alpha} \nabla_j \sigma^{\alpha} \nabla_k \sigma^{\beta} \nabla_l \sigma^{\beta})$$

$$+ \tfrac{1}{2} a^{-d} \sum_{\alpha} \int \mathrm{d}^d x \ln[1 - (\boldsymbol{\pi}^{\alpha})^2] \tag{2.8}$$

where

$$t^{-1} = (\rho_s \Lambda^{d-2})/T \qquad \rho_s = (z/d)[J]_{\mathrm{av}} a^{-(d-2)} \tag{2.9}$$

and

$$v_{ijkl} = 2\pi[\tfrac{1}{2} v_1(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) + v_2(1/d)\delta_{ij}\delta_{kl}] \tag{2.10}$$

with

$$v_1 = \frac{1}{4\pi}\frac{1}{(d+2)}\frac{[(\delta J)^2]_{\mathrm{av}}}{[J]_{\mathrm{av}}^2} a^2 \Lambda^{-2} \tag{2.11}$$

and

$$v_2 = \frac{1}{8\pi}\frac{d}{(d+2)}\frac{[(\delta J)^2]_{\mathrm{av}}}{[J]_{\mathrm{av}}^2} a^2 \Lambda^{-2}. \tag{2.12}$$

The last term in (2.8) arises as usual from the Jacobian (Pelcovitz and Nelson 1976) associated with the parametrisation of $\boldsymbol{S}^{\alpha}$ in terms of $\boldsymbol{\pi}^{\alpha}$. The ratio of v_1 to v_2 is $d/2$ if the exchange $J(\boldsymbol{x}, \boldsymbol{x}')$ is an independent random variable at each bond $\langle \boldsymbol{x}, \boldsymbol{x}' \rangle$. This ratio is modified if there is correlation in the exchanges on neighbouring bonds, for example if $[\delta J(\boldsymbol{x}_1, \boldsymbol{x}_2)\delta J(\boldsymbol{x}_3, \boldsymbol{x}_4)]_{\mathrm{av}}$ has one value if $\langle \boldsymbol{x}_1, \boldsymbol{x}_2 \rangle$ and $\langle \boldsymbol{x}_3, \boldsymbol{x}_4 \rangle$ are the same bond and another value if $\langle \boldsymbol{x}_1, \boldsymbol{x}_2 \rangle$ and $\langle \boldsymbol{x}_3, \boldsymbol{x}_4 \rangle$ are different bonds sharing a common site. In general, correlations in J could generate additional contributions to v_{ijkl} reflecting the point group symmetry of the lattice.

Simple power counting (as manifested by the powers of Λ in (2.9), (2.11), and (2.12)) shows that t is the most relevant potential near $t = 0$ and that higher-order cumulants of J are more irrelevant than the second-order cumulants giving rise to v_1 and v_2. We will retain only the most relevant terms shown in (2.8) for our renormalisation analysis, neglecting third- and higher-order cumulants of J. It should be noted that v_1 and v_2 are proportional to $[J]_{\mathrm{av}}^{-2}$ and are infinite when $[J]_{\mathrm{av}} = 0$. Therefore, fixed points with finite v_1 and v_2 correspond to non-zero $[J]_{\mathrm{av}}$ and thus not to spin-glass phases (Edwards and Anderson 1975, Binder and Young 1986).

3. Recursion relations

Momentum shell recursion relations for the potentials t, v_1 and v_2 can be obtained in

the usual way (Pelcovitz and Nelson 1976) to one-loop order. In the limit $n \to 0$, we find

$$\frac{dt(l)}{dl} = -\varepsilon t(l) + \frac{m-2}{2\pi} t^2(l) + [6v_1(l) + 2v_2(l)]t(l)$$

$$\frac{dv_1(l)}{dl} = -dv_1(l) + 23v_1^2(l) + 12v_1(l)v_2(l) + v_2^2(l) \tag{3.1}$$

$$\frac{dv_2(l)}{dl} = -dv_2(l) + 11v_2^2(l) + 14v_1(l)v_2 + 9v_2^2(l)$$

where $\varepsilon = d - 2$. We have not included temperature-dependent contributions in the recursion relations for v_1 and v_2. When $v_1 = v_2 = 0$, the equation for $t(l)$ reduces to that of a pure system which predicts the exponentially divergent correlations length of (1.5) for $\varepsilon = 0$. At a fixed point where $6v_1^* + 2v_2^*$ is non-zero and positive, the last term dominates over the second when $\varepsilon = 0$, leading to the power-law divergence of (1.3) with $\phi/\nu = 6v_1^* + 2v_2^*$. Thus if the v_1 and v_2 equations have a non-trivial fixed point then, the t equation produces the expected exponential to power-law crossover.

The fixed points of the v equations can be obtained by solving a cubic equation. The values of v_1 and v_2 and associated stability exponents are listed in table 1. The renormalisation group flows for v_1 and v_2 are shown in figure 2. Note that the pure system fixed point with $v_1 = v_2 = 0$ is globally stable in agreement with Murthy (1988). There are two twice-unstable fixed points in the unphysical second and fourth quadrants (v_1 and v_2 must be positive since they are proportional to the second cumulant or the variance of a random variable). There is one once-unstable fixed point in the physical first quadrant which we identify with the percolation fixed point. The inverse of the positive exponent is the percolation correlation length exponent

$$\nu_p = \lambda^{-1} = \tfrac{1}{2} \tag{3.2}$$

in this approximation. It differs significantly from the exact result (den Nijs 1979, Nienhuis 1982) of $\frac{4}{3}$, indicating that neglect of higher cumulants of $[J]$ is not a quantitatively good approximation.

To study crossover near $t = 0$, we integrate the recursion relations for $t(l)$:

$$t(l) = \frac{Q(l)t(0)}{1 - [(m-2)/(2\pi)] \int_0^l Q(l)\, dl} \tag{3.3}$$

where

$$Q(l) = \exp\left(\int_0^l [6v_1(l) + 2v_2(l)]\, dl \right). \tag{3.4}$$

At the percolation fixed point $Q(l) = \exp[(6v_1^* + 2v_2^*)l]$ and

$$t(l) \sim \exp[(\phi/\nu_p)l^*]t(0) = \exp[(6v_1^* + 2v_2^*)l^*]t(0) \tag{3.5}$$

Table 1. Fixed points and stability exponents.

Fixed point	v_1^*	v_2^*	λ_1	λ_2
0	0	0	−2.0	−2.0
1	0.038 592	0.079 166	2.0	−0.769 2355
2	−0.289 051	1.021 527	3.256 027	2.0
3	0.289 106	−0.427 737	2.028 931	2.0

so that

$$e^{l^*} = \xi(t)/\xi_0 = (T/[J]_{av})^{-\phi} \tag{3.6}$$

where

$$\phi = \nu_p(6v_1^* + 2v_2^*) = 0.194\,94$$

and ξ_0 is the bare correlation length. The expected power law in temperature dependence of the correlation length at the percolation threshold is reproduced by our analysis, but again the numerical value of the exponent ϕ differs significantly from the accepted value of 1.296.

To simplify our analysis of these equations, we will assume that $v_1(0)/v_2(0) = v_1^*/v_2^* \equiv \rho^*$, so that flow is along the line connecting the percolation fixed point to the origin. Corrections arising from deviations of $v_1(l)/v_2(l)$ from ρ^* can easily be treated and will not significantly alter our results. In this case, the equation for v_1 can be integrated exactly

$$v_1(l) = \frac{e^{-2l}v_1(0)}{1-[v_1(0)/v_1^*](1-e^{-2l})} \tag{3.7}$$

from which we obtain

$$Q(l) = \{1-[v_1(0)/v_1^*](1-e^{-2l})\}^{-\phi}. \tag{3.8}$$

To obtain $\xi(l)$ for $v_1 > v_1^*$ ($p > p_c$), note that $\lim_{l\to\infty} Q(l) \to [1-(v_1(0)/v_1^*)]^{-\phi}$, so that

$$t(l) \sim \frac{Q(\infty)t(0)}{1-[(m-2)/(2\pi)]t(0)Q(\infty)l}. \tag{3.9}$$

Then, choosing the matching point so that $t(l^*) = 2\pi/(m-2)$, we obtain

$$\begin{aligned} e^{l^*} &= \xi/\xi_0 \\ &= \exp[(2\pi t(0)/Q(\infty))] - 1] \equiv \exp[(2\pi\rho_s(p)/T) - 1] \end{aligned} \tag{3.10}$$

where

$$\rho_s(p)/\rho_s(0) = [1-(v_1(0)/v_1^*)]^{\phi} = (|p-p_c|/p_c)^{\phi}. \tag{3.11}$$

Thus ξ diverges exponentially with a renormalised exponent determined by the diluted spin wave stiffness that tends to zero as $|p-p_c|^{\phi}$, as required. The spin wave stiffness can be calculated directly via

$$\rho_s(p) = \lim_{T,q\to 0} [(T/q^2)G^{-1}(q)] \tag{3.12}$$

where $G(q)$ is the Fourier transform of the correlation function $\langle \boldsymbol{\pi}(x) \cdot \boldsymbol{\pi}(x)\rangle$. The result obtained in this way agrees with that of (3.11) to lowest order in an expansion in ϕ and $v_1(0)/v_1^*$. Exact agreement should not be expected because of the truncation procedure we have used.

4. Summary and discussion

We have used a replicated nonlinear sigma model to study the two-dimensional randomly diluted Heisenberg model with O_m symmetry for $m > 2$. In the truncated space of the most relevant of the potentials describing disorder, we found a once-unstable zero-temperature fixed point describing the percolation critical point at $p = p_c$. Renormalisation group recursion relations predict exponential divergence with temperature of the order parameter correlation length for $p > p_c$ and power-law divergence

at $p = p_c$. The numerical values of the percolation correlation length exponent ν_p and the thermal crossover exponent ϕ are, however, in poor agreement with the accepted values.

We have also investigated the randomly diluted quantum Heisenberg antiferromagnet using the techniques outlined in this paper. As discussed by Chakravarty *et al* (1988), the RVB fixed point is expected to be unstable with respect to randomness. In the one-loop truncated scheme we use, this fixed point is unfortunately stable, and the quantum percolation fixed point expected for this model (Raghavan and Mattis 1981, Harris, 1982, Shapir *et al* 1982) does not appear.

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